

Lagrangian view of time irreversibility of fluid turbulence

Haitao Xu,^{1,2,*} Alain Pumir,^{3,2} and Eberhard Bodenschatz^{2,4,5}

¹Center for Combustion Energy and Department of Thermal Engineering, Tsinghua University, Beijing 100084, China

²Max Planck Institute for Dynamics & Self-Organization (MPIDS), D-37077 Göttingen, Germany

³Ecole Normale Supérieure de Lyon, F-69007 Lyon, France

⁴Institute for Nonlinear Dynamics, University of Göttingen, D-37077 Göttingen, Germany

⁵Laboratory of Atomic and Solid State Physics and Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, NY 14853, USA

A turbulent flow is maintained by an external supply of kinetic energy, which is eventually dissipated into heat at steep velocity gradients. The scale at which energy is supplied greatly differs from the scale at which energy is dissipated, the more so as the turbulent intensity (the Reynolds number) is larger. The resulting energy flux over the range of scales, intermediate between energy injection and dissipation, acts as a source of time irreversibility. As it is now possible to follow accurately fluid particles in a turbulent flow field, both from laboratory experiments and from numerical simulations, a natural question arises: how do we detect time irreversibility from these Lagrangian data? Here we discuss recent results concerning this problem. For Lagrangian statistics involving more than one fluid particle, the distance between fluid particles introduces an intrinsic length scale into the problem. The evolution of quantities dependent on the relative motion between these fluid particles, including the kinetic energy in the relative motion, or the configuration of an initially isotropic structure can be related to the equal-time correlation functions of the velocity field, and is therefore sensitive to the energy flux through scales, hence to the irreversibility of the flow. In contrast, for single-particle Lagrangian statistics, the most often studied velocity structure functions cannot distinguish the “arrow of time”. Recent observations from experimental and numerical simulation data, however, show that the change of kinetic energy following the particle motion, is sensitive to time-reversal. We end the survey with a brief discussion of the implication of this line of work.

PACS numbers: 47.27.Gs, 47.27.Jv, 47.80.Fg

I. INTRODUCTION

Flowing fluids are ubiquitous in many natural and industrial situations. The running of water out of the faucet in our kitchen or the intimidating roar of a destructive hurricane provide examples involving the two most abundant fluids on earth: water and air. From elementary physical principles, the description of fluid motions is based on the competition between the inertia of fluid particles, and the diffusion of momentum by viscosity. As a result, the physical properties of the flow are characterized by the dimensionless Reynolds number $Re = UL/\nu$, where L and U are the typical length and velocity scales of the flow and ν is the kinematic viscosity of the fluid. The Reynolds number can be regarded as the ratio of the viscous time scale L^2/ν and the flow time scale L/U and therefore measures the relative importance of the inertial effect, which tends to drive the flow to become unstable, and the damping by the viscosity. For a flow at small Reynolds number, viscous diffusion is fast, so viscosity damps out flow disturbances. This situation is referred to as “laminar”. In such flows, energy dissipation transfers mechanical energy into heat, which can be readily seen from the few available exact solutions of the Navier-Stokes equations [1]. On the other hand, when the Reynolds number is large, the inertial effects dominate and the flow appears to be much more irregular, even in the absence of any externally imposed time-dependence.

Such flows are called “turbulent”. In three dimensional situations, turbulent flows are “rough”, in the sense that they develop strong variations of the velocity field over very small scales, or equivalently, very large velocity gradients. In these regions, the viscosity is important. The irregular nature of turbulent flows leads to a fast and seemingly erratic motion of small particles transported by turbulence. The work reviewed here shows that the fundamental properties of the flow, such as the irreversibility induced by the energy dissipation, manifest themselves in the motion of small particles.

Most macroscopic flows in nature and technology are turbulent. This is a consequence of the very small values of the viscosities of the most common fluids, such as air and water, which leads to large Reynolds numbers, even at modest length and velocity scales. For example, an adult walking at a moderate pace creates an air flow with a Reynolds number of approximately 5×10^4 around him/her, and the flow of tap water in our kitchen can easily reach a Reynolds number of 10^4 , both of which are well in the turbulent flow regime.

Because of their rapid erratic motion, turbulent flows strongly enhance mixing. This we know well from stirring water in order to dissolve sugar added in it. The same principle we apply when we rapidly mix fuel and air in combustion engines. This of course comes at the expense of energy: The intense, erratic turbulent flow needs to be maintained by external driving or pumping. Sometimes one would like to avoid turbulence. Examples can be found in fluid drag on trains, cars, ships and airplanes, or in pipe flows, where a much larger pressure drop is required to pump a turbulent flow through a pipe than a laminar flow at the same flow rate. Thus a better understanding of turbulence and turbulent flows could allow

*Electronic address: hxu@tsinghua.edu.cn

us, on the one hand, to mix fluids more efficiently and, on the other hand, to reduce the drag in technical applications.

From the point of view of fundamental physics, turbulence is an emblematic example of non-equilibrium systems, whose description is notoriously challenging. In particular, novel concepts to master the underlying complexity of turbulent flows are yet to be discovered. These concepts could then also provide deeper insights into other non-equilibrium problems. One of the unifying concepts to address these problems is the breaking of detailed balance. Irreversibility in turbulence implies that the transition probabilities from a state A to a state B, and from the state B to the state A are *not* equal, contrary to what happens in equilibrium systems in statistical mechanics [2–4].

Like other out-of-equilibrium systems, turbulent fluid flows are irreversible. The kinetic energy of the flow is always dissipated and a constant supply of kinetic energy is necessary to maintain a turbulent flow. Whereas it is a simple matter to understand dissipation (hence irreversibility) in laminar flows, it is much more challenging to identify the features of the turbulent flow motions that reveal irreversibility. In turbulent flows, the nonlinearity plays a crucial role, and leads to chaotic motion, with subtle statistical properties. A specific property of turbulent flows is that the scales at which energy is supplied, either from external forcing or from flow instabilities, are vastly different from the scales at which the energy is dissipated. In three-dimensional (3D) flows, the energy injection is at large scales while the viscous dissipation dominates at small-scales [5–7]. In two-dimensional (2D) flows, the energy is supplied at small scales and then dissipated by friction at large scales [8, 9]. The scale-separation between the energy injection and energy dissipation implies that there is, on average, a flux of kinetic energy through spatial scales between the energy injection scale and the energy dissipation scale, i.e., in the so-called inertial range, and the direction of this average energy flux cannot be reversed. This is the underlying reason why turbulent flows are irreversible. There are fundamental differences between fluctuations in steady-state turbulence and fluctuations in equilibrium systems [10].

Remarkably, the celebrated Kolmogorov’s 4/5-law shows that in statistically stationary turbulent flows, the energy flux can be measured from a single snapshot of the velocity field [11, 12], without any explicit reference to its temporal evolution. The 4/5-law can be generalized to both 2D and 3D cases as (see e.g. Ref. [13])

$$\left\langle \left[\left(\mathbf{u}(\mathbf{x} + \mathbf{r}, t) - \mathbf{u}(\mathbf{x}, t) \right) \cdot \frac{\mathbf{r}}{r} \right]^3 \right\rangle = -\frac{12}{d(d+2)}\epsilon r, \quad (1)$$

where $\mathbf{u}(\mathbf{x}, t)$ is the velocity field, \mathbf{r} is the separation vector whose magnitude is r , d is the spatial dimension, and ϵ is the turbulent energy flux, defined as a signed quantity, with the convention that $\epsilon > 0$ when energy is transferred towards small scales and $\epsilon < 0$ for energy flux toward large scales. The symbol $\langle \cdot \rangle$ refers to an *ensemble average*, obtained by averaging over all possible flow realizations. For statistically homogeneous flows, it is the same as averaging over time t (under the assumption of ergodicity), or over space \mathbf{x} . Equation (1) establishes that the third moment of the longitudinal

velocity increments, $\delta_r u \equiv \left(\mathbf{u}(\mathbf{x} + \mathbf{r}, t) - \mathbf{u}(\mathbf{x}, t) \right) \cdot (\mathbf{r}/r)$, differs from zero, which is a consequence of the existence of a flux of energy through scales. This equation is valid when the distance r between the two measured velocities is in the inertial range, i.e., when $\ell_s \ll r \ll \ell_L$, where ℓ_s and ℓ_L are the smallest and the largest length scales of the flow whose physical meanings depend on the spatial dimension of the flow. For 3D turbulence, $\ell_s = \eta$, the well-known Kolmogorov scale given by the balance between the viscous dissipation and the energy flux ϵ (remember that ϵ can be either positive or negative, depending on the nature of the transfer of energy), and ℓ_L is the forcing scale. In the 3D turbulence case, $\epsilon > 0$ and the flux is from the large to the small scales and it is called *direct energy cascade*. On the other hand, for 2D turbulence, the energy flux is negative: $\epsilon < 0$ and it is called *inverse energy cascade* because the kinetic energy is from the forcing scale ℓ_f , which is the small scale ℓ_s , to the large scale ℓ_L that is determined by the balance between large-scale friction and the energy flux.

We stress that in Eq. (1), both $\mathbf{u}(\mathbf{x} + \mathbf{r})$ and $\mathbf{u}(\mathbf{x})$ are measured at the *same* time t , i.e., in principle using only a single “snapshot” of the velocity field. This property leads to the following paradoxical situation. Consider a sequence of velocity fields, which correspond to a solution of the Navier-Stokes equation. According to Eq. 1, the precise order of the snapshots is immaterial in the determination of the rate of energy dissipation. In particular, simply reversing the order of the sequence, i.e., changing $t \rightarrow -t$ does not seem to affect the determination of the energy dissipation rate, which is obtained from the individual velocity field (Eulerian statistics) alone. Thus, we can measure the energy flux correctly but cannot detect the time-irreversibility, although the energy flux is the cause of the irreversibility.

To understand this paradoxical situation, we investigate here the motion of fluid particles moving with the flow, i.e., Lagrangian statistics. In the last two decades, significant advances in measurement techniques and computing technologies made it possible to obtain well-resolved Lagrangian statistics at high Reynolds numbers in both experiments [14–20] and numerical simulations [21–25]. Unexpectedly, as explained in Section III A, the Lagrangian structure functions turn out to be completely insensitive to the fundamental irreversibility of the flow [26], thus calling for new concepts and ideas. In the following, we review briefly these recent results, with the focus on the relation to the irreversibility of turbulence.

II. MULTI-PARTICLE STATISTICS: THE ROLE OF ENERGY FLUX

We start with multi-particle Lagrangian statistics, i.e., by following more than one fluid particle in a turbulent flow. To this end, we consider elementary sets of particles, and study the flow perceived by the particles, and its dependence on the characteristic distance between the particles. We show that when the inter-particle separations are in the inertial range, the energy flux through scales results in a measurable difference

between the statistical properties of the motion forward and backward in time.

A. Relative dispersion and energy considerations

In a turbulent flow, two fluid particles are, on average, moving away from each other. This can be quantified by measuring the mean squared stance between the two particles, $\langle R^2(t) \rangle \equiv \langle |\mathbf{X}_1(t) - \mathbf{X}_2(t)|^2 \rangle_{R_0}$ where $\mathbf{X}_1(t)$ and $\mathbf{X}_2(t)$ are the positions of particles 1 and 2 at time t . The subscript R_0 in the definition of the mean squared distance between two particles, $\langle R^2(t) \rangle$, refers to the imposed condition that at $t = 0$ the initial distance $R(0) = |\mathbf{X}_1(0) - \mathbf{X}_2(0)|$ is equal to R_0 .

How $\langle R^2(t) \rangle$ changes with time quantifies the relative dispersion of a particle pair. The problem has been extensively studied since the pioneering work by Richardson [27], who observed that in turbulent flows, in a meteorological context, the mean squared distance between two particles grows with time as t^3 , i.e., $\langle R^2(t) \rangle \propto t^3$. Paradoxically, although Richardson t^3 law is easy to justify theoretically, it has proven very difficult to observe this regime of turbulent dispersion in any other well-controlled laboratory flows, or in direct numerical simulations, and many investigations have been devoted to this phenomena [14, 18, 23, 28–34].

In general, the separation $\langle R^2(t) \rangle$ can be expressed as

$$\langle R^2(t) \rangle = R_0^2 + 2 \int_0^t \int_0^{t'} \langle \delta \mathbf{u}(t') \cdot \delta \mathbf{u}(t'') \rangle dt'' dt', \quad (2)$$

where $\delta \mathbf{u}(t) \equiv \mathbf{u}_1(t) - \mathbf{u}_2(t)$ is the relative velocity between the two fluid particles. Equation (2) thus relates the relative dispersion of two particles to the Lagrangian correlation of relative velocity, $\langle \delta \mathbf{u}(0) \delta \mathbf{u}(\tau) \rangle$. In fact, considering $\langle \delta \mathbf{u}(0) \delta \mathbf{u}(\tau) \rangle$ in the limits of small values of τ ($\tau \rightarrow 0$), and τ large compared to the velocity correlation time ($\tau \rightarrow \infty$), leads to interesting information on the time-dependence of $\langle R^2(t) \rangle$. When $\tau \rightarrow \infty$, the relative velocity $\delta \mathbf{u}(\tau)$ becomes independent of its initial value $\delta \mathbf{u}(0)$, so the double integral in Eq. (2) is linear in t . This corresponds to a diffusion-like regime that is similar to the turbulent diffusion of single particles first discussed in the seminal work of G. I. Taylor [35]. In the opposite limit of very short times, $\tau \rightarrow 0$, we can expand the integrand in Eq. (2) at $t = 0$ in power series of t and integrate to obtain:

$$\langle R^2(t) \rangle = R_0^2 + \langle \delta u^2(0) \rangle t^2 + \langle \delta \mathbf{u}(0) \cdot \delta \mathbf{a}(0) \rangle t^3 + \mathcal{O}(t^4), \quad (3)$$

where $\delta u^2(0)$ is a simplified notation for $\delta \mathbf{u}(0) \cdot \delta \mathbf{u}(0)$ and $\delta \mathbf{a}(t) \equiv \mathbf{a}_1(t) - \mathbf{a}_2(t)$ is the relative acceleration between the two particles. Equation (3) shows that as long as the initial separation R_0 is non-zero, the initial velocity difference does not vanish and $\langle R^2(t) \rangle$ is dominated by the t^2 term at very small times. The Richardson t^3 regime, therefore, can only exist for some intermediate time t after marking the two particles [36–38]. Furthermore, we note that $\delta \mathbf{u}(t) \cdot \delta \mathbf{a}(t) = \frac{d}{dt} \left[\frac{1}{2} \delta u^2(t) \right]$ is the rate of change of the kinetic energy in the relative motion between the two particles.

It has been shown that for separation R_0 in the inertial range, this rate of kinetic energy change is related to the turbulent energy cascade through scales [13, 14, 39]:

$$\langle \delta \mathbf{u} \cdot \delta \mathbf{a} \rangle_{R_0} = \frac{d}{dt} \left\langle \frac{1}{2} \delta u^2 \right\rangle_{R_0} = -2\epsilon, \quad (4)$$

in which the averaging is taking over all particle pairs separated by a distance R_0 in the inertial range. As we mentioned before, the energy flux ϵ is positive for 3D flows. Hence in 3D turbulence the kinetic energy in the relative motion conditioned on a given separation R_0 between particles initially *decreases*. It increases at later times, consistent with the faster than t^2 increase of $\langle R^2(t) \rangle$ at later times. This unexpected consequence of the energy cascade has been confirmed, both in numerical simulations [39] and in Lagrangian particle tracking experiments [40]. Note that substituting Eq. (4) into Eq. (3) gives a negative t^3 term for $\langle R^2(t) \rangle$, which should not be confused with the positive coefficient in the Richardson dispersion law expected at later times.

In the problem of mixing of a passive scalar, a proper modeling of the fluctuations of concentration rests on understanding how two fluid particles arrive at a given distance apart, or in other words, how $\langle R^2(t) \rangle$ changes when $t < 0$. This amounts to tracking the motion of particles backward in time. For backward dispersion, it is also expected that a Richardson-like regime exists, i.e., $\langle R^2(-t) \rangle \propto t^3$ for intermediate time t , but with a larger coefficient, which means that backward dispersion is faster than forward dispersion [41–43]. An interesting observation is that for incompressible Navier-Stokes turbulence, both Eq. (2) and Eq. (3) are also valid for $t < 0$. We note that in 3D turbulence, the t^3 term in Eq. (3) is *positive* for $t < 0$, which implies that even for short times, backward dispersion is faster than forward dispersion. This is a consequence of the fact that the kinetic energy of the relative motion of particle pairs, followed backwards in time, *increases* at short times, contrary to what happens when following particle motion forward in time.

This property is a manifestation of time irreversibility of turbulence. In principle, it can be used to detect the “arrow-of-time”, while following many particle trajectories in a turbulent flow [44]. This manifestation of time-irreversibility in the relative dispersion between two fluid particles, $d\langle R^2 \rangle/dt$, ultimately rests on the relation between the turbulent energy cascade and the rate of energy change in the relative motion expressed by Eq. (4). This means that if we know the derivative of the kinetic energy in the relative motion, $d(\frac{1}{2} \delta u^2)/dt$, or the velocity field and its time derivative $d\delta \mathbf{u}/dt$, then we can also detect the “arrow of time”. On the same basis, other Lagrangian quantities that combine both relative velocity and separation R can also be formed with the property that their time derivatives are sensitive to whether the “arrow of time” is flipped or not [45]. A further observation is that for other flows that do not satisfy incompressible Navier-Stokes equations, Eq. (4) might not remain the same for $t > 0$ and $t < 0$, i.e., there could be an anomaly in Lagrangian velocity statistics. For example, it has been shown that for the compressible Burgers equation, taking $d(\delta u^2)/dt$ at $t = 0$ from the $t < 0$ and $t > 0$ side give different values, which is due to the for-

mation of shocks in Burgers turbulence when time is running forward [46]. This is clearly a stronger manifestation of time irreversibility.

While Eq. (4) has been derived for both 2D and 3D turbulence, its validity has so far been verified numerically and experimentally only for 3D turbulence, but not for 2D turbulence. The physics of energy cascade is completely different in 2D turbulence [8, 9, 13], compared to that in 3D turbulence. In particular, the energy flux in 2D is towards larger scales, and consequently the kinetic energy in the relative motion between fluid particle pairs is expected to increase initially. Confronting this prediction with numerical and experimental data could be an interesting and important work for the future.

B. Shape deformation and structure of the flow

While the previous section was devoted to the relative motion between two particles, we now turn to the Lagrangian statistics involving more than two particles. The description of a set of points requires not only a size, such as the mean distance between the particles, but also extra variables describing the shape of the set of points. The shape evolution provides interesting information on the local (topological) structure of the turbulent flow, which cannot be obtained from the study of the mean separation between pairs of particles alone. To explore the flow topology, one needs to follow at least 3 particles in a 2D flow and 4 particles in a 3D flow. It has been observed that the evolution of initially isotropic objects (equilateral triangles or regular tetrahedra) in turbulent flows differs from that in a Gaussian velocity field. Qualitatively, the shapes obtained in a turbulent flow are more elongated at intermediate times than expected by using a flow with Gaussian statistics, or before all particles are widely separated so their velocities become independent [22, 24, 25, 40, 47]. To see how this is related to flow topology, one can define an effective local velocity gradient \mathbf{M} perceived by the set of particles [20, 39, 48, 49]:

$$\mathbf{M} = \mathbf{g}^{-1} \mathbf{W}, \quad (5)$$

where the matrices \mathbf{g} and \mathbf{W} are defined as

$$g_{ij} = \sum_{\alpha=1}^N x_i'^{\alpha} x_j'^{\alpha} \quad (6)$$

and

$$W_{ij} = \sum_{\alpha=1}^N x_i'^{\alpha} u_j'^{\alpha} \quad (7)$$

where N is the total number of particles in the set used to define the perceived velocity gradient \mathbf{M} ,

$$\mathbf{x}'^{\alpha} = \mathbf{x}^{\alpha} - \frac{1}{N} \sum_{\alpha=1}^N \mathbf{x}^{\alpha} \quad (8)$$

and

$$\mathbf{u}'^{\alpha} = \mathbf{u}^{\alpha} - \frac{1}{N} \sum_{\alpha=1}^N \mathbf{u}^{\alpha} \quad (9)$$

are the position and velocity of particle α relative to the center of the particle set. It is easy to show that the perceived velocity gradient \mathbf{M} given by Eq. (5) is the least square fit of a linear velocity field from the velocities at the particle positions. When the separations between the particles are very small, in the range where viscous effects dominate, the perceived velocity gradient \mathbf{M} given by Eq. (5) recovers the true velocity gradient. When the particle separations are in the inertial range, \mathbf{M} probes the inertial range dynamics, which is the main motivation to study \mathbf{M} [48]. Other effective local velocity gradients similar to \mathbf{M} have been proposed and the information on flow topology obtained are also comparable [17, 50].

To probe the flow topology, it is helpful to decompose the velocity gradient \mathbf{M} as a sum of a symmetrical part, $\mathbf{S} = (\mathbf{M} + \mathbf{M}^T)/2$, which represents the rate of strain (local stretching or compression) of the flow, and an anti-symmetrical part, $\mathbf{\Omega} = (\mathbf{M} - \mathbf{M}^T)/2$, which represents the local rotation (by construction, $\mathbf{S} + \mathbf{\Omega} = \mathbf{M}$). This decomposition is in fact unique. The strain \mathbf{S} and the rotation $\mathbf{\Omega}$ interact with each other, which forms the rich dynamics of turbulent flows. In particular, on average the local rotation rate is constantly amplified because of the action of the strain, a phenomenon called “vortex stretching”, which is eventually compensated by the viscous dissipation. Early studies of the true velocity gradient in turbulent flows have revealed that the stretching of the vorticity is closely tied to the statistics of the eigenvalues of the strain \mathbf{S} . Namely, among the three eigenvalues, which are all real because \mathbf{S} is symmetric by definition, the intermediate eigenvalue is predominately positive [51, 52]. This has been verified in numerical simulations and experiments [53, 54] and has stimulated further studies on the dynamics of velocity gradients [55–57]. A natural expectation from vortex stretching is that the vorticity vector would be preferentially aligned with the direction corresponding to the largest eigenvalue of the strain, which represents the strongest stretching. On the other hand, numerical and experimental data show that at any given instant, vorticity is preferentially aligned with the intermediate eigenvalue of the strain, which corresponds to rather mild stretching [54, 58, 59]. This interesting observation has been studied extensively in subsequent research (see [60] for a detailed discussion).

When the size of the particle cluster used to obtain \mathbf{M} from Eq. (5) is larger than the viscous range, \mathbf{M} differs from the true velocity gradient and provides a way to probe the flow property in the inertial range of scales. It has been observed that \mathbf{M} obtained in this way shares qualitatively many properties of the true velocity gradient. In particular, the intermediate eigenvalue of the rate of strain \mathbf{S} is predominately positive and instantaneously the vorticity is aligned with this intermediate eigenvalue [48, 49, 61]. These properties of \mathbf{M} , which are closely related to the inertial range dynamics and hence to energy cascade, are expected to lead to irreversibility in Lagrangian multi-particle statistics. For example, the short-time deformation of an initially isotropic tetrahedron formed by four fluid particles is governed by the eigenvalues of the perceived rate of strain. Therefore, the non-zero average of the intermediate eigenvalue of \mathbf{S} implies that the shape evolution

of a tetrahedron differs when followed forward or backward in time [44]. The Lagrangian view also shows that the perceived vorticity vector indeed tends to align with the largest eigenvalue of \mathbf{S} , in the sense that the vorticity vector turns to the initially strongest stretching direction, but with a time delay such that at any given instant the vorticity is observed to preferentially align with the intermediate eigenvalue [20]. This property is also found for the true velocity gradient [49, 62]. The observed alignment process of vorticity with the rate of strain at a given time will completely differ when following flow trajectories backward in time. We note that when the length R_0 characteristic of the set of points is in the inertial range of scales, the dynamical processes are essentially self-similar. This can be seen by rescaling time by the time scale $t_0 = (R_0^2/\epsilon)^{1/3}$, which can be viewed as the eddy-turnover time at scale R_0 [20, 44, 49].

A different but related interesting question is how rigid particles with given shapes see the turbulent flow. For small rod-like particles, it is surprising to observe that they align with local vorticity and hence preferentially with the intermediate eigenvector of the rate of strain [63–65]. New results are available concerning the coupling between translation and rotation of neutrally buoyant particles with other shapes, such as large spheres [66, 67], ellipsoids [68–70], or other anisotropic shapes [71]. It would be very interesting to find how their dynamics are related to the irreversibility of the flow.

In summary, we note that for multi-particle Lagrangian statistics, the distance between particles defines a natural length scale of the problem and the energy cascade process in turbulence inherently causes the observed irreversibility. Stochastic models have been widely used to describe the multi-particle dispersion process and many aspects of the observed multi-particle statistics can be recovered by these models [72, 73]. As almost all these models have time-reversible dynamics built in, one should be cautious not to push these models beyond the range in which they are valid.

III. SINGLE-PARTICLE STATISTICS

In situations involving several particles, we can naturally introduce one length scale (or more) in the problem, therefore permitting to establish a relation with the Eulerian correlation functions of the velocity field, and hence the energy flux or energy dissipation. In contrast, following only one fluid particle in a turbulent flow does not give rise to an unambiguous identification of a length scale. For this reason, new ideas and concepts are needed in order to understand the flow properties from the statistics of single particle trajectories only. Recent progress provides new insights into these interesting questions.

A. Velocity structure functions

The most studied single-particle statistic quantities are the Lagrangian velocity structure functions, i.e., the moments of

the velocity increments following a fluid particle:

$$S_n(\tau) = \langle (\delta_\tau u)^n \rangle \equiv \langle [u(t+\tau) - u(t)]^n \rangle, \quad (10)$$

where $u(t)$ is one component of the particle velocity at time t along a direction, being understood that for homogeneous and isotropic turbulence, the choice of the component does not matter. By analogy with the Eulerian velocity increments, one may surmise that the Lagrangian velocity increments $\delta_\tau u$ depend on the turbulent energy dissipation rate ϵ . Furthermore, if the time lag τ is much larger than the viscous time scale but smaller than the largest time scale of the flow, it is tempting to postulate that the statistics of the velocity increments are universal and independent of viscosity. Simple dimensional analysis then leads to the scaling $\delta_\tau u \sim (\epsilon\tau)^{1/2}$ and hence $S_n(\tau) \sim (\epsilon\tau)^{n/2}$ [1, 32, 38, 74]. Available experimental and numerical data show that the dependence of $S_n(\tau)$ on τ has very little to do with the expected scaling behavior [16, 75, 76]. Various theories have been proposed to explain the observed deviations [32, 77, 78], with the multifractal model being the most popular [75, 79, 80] (see also a recent summary in Ref. [81]).

Among the Lagrangian structure functions, the second order, obtained by taking $n = 2$ in Eq. (10), is of special interest because according to the dimensional argument it is proportional to the energy dissipation rate ϵ itself, so the average in Eq. (10) is not affected by the strong fluctuations in ϵ , which is known to lead to corrections to scaling in the case of the spatial structure functions (intermittency corrections) [6, 38, 82]. Based on these considerations, the scaling $S_2(\tau) \propto \epsilon\tau$ is expected to be exact, just as the linear scaling of the third-order Eulerian velocity structure function predicted by Eq. (1) (the 4/5-law). This expectation is summarized by the following relation:

$$S_2(\tau) = \langle (\delta_\tau u)^2 \rangle = C_0 \epsilon \tau, \quad (11)$$

where C_0 is expected to be a universal constant of order unity [32, 38, 74]. The best available data, from state-of-the-art experiments and numerical simulations, however, does *not* support the scaling suggested by Eq. (11) [32, 83]. If anything, the values of C_0 observed for 3D turbulence are found to increase with the Reynolds number of the flow and are approximately 7 for the highest Reynolds numbers measured so far [32, 83]. In the case of 2D turbulence, where Eq. (11) is also expected to hold, the values of C_0 increase much faster with the Reynolds number. The results of numerical simulations at the highest available resolution suggest values of the order $\sim \mathcal{O}(10^2)$, without any indication of saturation [26]. This casts a serious doubt on the scaling predicted by using dimensional arguments.

In fact, the assumption that the statistics of velocity increments following a fluid particle, $\delta_\tau u$, depend on ϵ , the energy flux through spatial scales, is questionable. That assumption is directly inspired from Kolmogorov's hypotheses on the statistics of Eulerian velocity increments. As already noticed, establishing a connection between Lagrangian and Eulerian statistics requires the introduction of a length scale into the structure functions $S_n(\tau)$, which is achieved by assuming that

$\tau\delta_\tau u$ is equivalent to the separation r in Eulerian statistics and that $\delta_\tau u$ scales as the Eulerian velocity difference $\delta_r u$. Substituting $\delta_r u \sim \delta_\tau u$ and $r \sim \tau\delta_\tau u$ into Eq. (1) leads to $\langle(\delta_\tau u)^2\rangle \sim \epsilon\tau$, hence to a formal justification of Eq. (11).

The formal analogy between the Eulerian velocity increments, $\delta_r u$, and the Lagrangian velocity increments $\delta_\tau u$, through the use of $r \approx \tau\delta_\tau u$ should, however, be taken very carefully. The Eulerian statistics of $\delta_r u$ are mostly determined by turbulent eddies of size r . When estimating the Lagrangian increment $\delta_\tau u$ using the Eulerian increment $\delta_r u$ with $r \approx \tau\delta_\tau u$, it should be noticed that the time τ necessary for a particle to travel up to r , $\tau \sim r/\delta_\tau u \sim (\delta_\tau u)^2/\epsilon$, is in fact the life time of an eddy of size r . That is to say, for a fluid particle to move a distance r , its velocities at the start, $u(t)$, and at the end, $u(t + \tau)$, are unlikely to be the result of the same eddy of size r . This essential dissimilarity between $\delta_r u$ and $\delta_\tau u$ leads to very different properties between Eulerian and Lagrangian statistics. More generally, this feature highlights statistically stationary turbulence, as an ultimate example of non-equilibrium steady state, far from equilibrium [10].

These, and other considerations led Falkovich et al. [26] to question the validity of Eq. (11). They pointed out that the statistics of $\delta_\tau u$, including all Lagrangian velocity structure functions, are symmetric under the transformation of $t \rightarrow -t$, therefore being unable to pick up the fundamental time-irreversibility of the flow. Therefore, there is no fundamental reason to relate the statistics of $\delta_\tau u$ to the energy flux, which is the cause of the time-irreversibility of turbulent flows.

In summary, there is strong motivation to consider other Lagrangian statistics that reveal the irreversible nature of turbulent flows. That is the topic we cover in the next subsection.

B. Kinetic energy increments and instantaneous power

An interesting recent discovery is that the change of kinetic energy following individual fluid particles can be used to detect the “arrow of time”. In Ref. [84], it was observed in experiments and numerical simulations that the third moments of the kinetic energy change, $\delta_\tau W \equiv [u^2(\tau) - u^2(0)]/2$:

$$\langle(\delta_\tau W)^3\rangle \equiv \langle[u^2(t + \tau)/2 - u^2(t)/2]^3\rangle \quad (12)$$

are negative for time lags τ positive, but smaller than the velocity correlation time (the largest time scale of the flow). This implies that the probability distribution of the instantaneous power, $p = \lim_{\tau \rightarrow 0} \delta_\tau W/\tau = \mathbf{u} \cdot \mathbf{a}$, is negatively skewed. The origin of this skewness can be traced back to the observed tendency of fluid particles to gain kinetic energy slowly, but lose it more suddenly. This provides a way to identify the arrow of time, as flipping $t \rightarrow -t$ would lead to the exact opposite: particles would gain energy faster than they dissipate it. The negative skewness of the distribution of p was observed for both 2D and 3D turbulence (at least for 2D turbulent flows that were agitated with forces short-correlated in time), i.e., independent of whether energy flows towards larger or smaller scales. From the more general point of view of energy exchange, kinetic energy is dissipated into heat in an irreversible

way in turbulent flows, both in 2D and 3D. In that sense, the qualitative similarity between the statistics of p and $\delta_\tau W$ in both 2D and 3D turbulences, once expressed in terms of the energy dissipation, ϵ , may not be so surprising. Available data support that the moments of p/ϵ follow, to a good approximation, a power law dependence on the Reynolds number of the flow, with an exponent independent of the spatial dimension. This suggests that the third moment $-\langle p^3 \rangle/\epsilon^3$ can be used as a measure of irreversibility. Moreover, the scaling of the third moment $-\langle p^3 \rangle/\epsilon^3$ can be qualitatively explained by assuming that $-\langle p^3 \rangle/\epsilon^3$ is dominated by the extreme events of negative p with large magnitudes, i.e., events when fluid particles lose kinetic energy very rapidly, an argument pictorially alluding to “flight-and-crash” events [84]. This skewed distribution of p also manifests itself in the negative skewness of the kinetic energy change associated with single velocity component in a 3D turbulent flow [85], and in the negative skewness of the longitudinal Lagrangian velocity increments [86].

While the skewness of the instantaneous power p is negative for both 2D and 3D turbulence, and in this sense, seems to be insensitive to the very different physical mechanisms of cascade in these two cases, one may nevertheless ask which quantity reflects the difference in the dynamics in 2D and 3D flows. To answer that question, one can decompose the instantaneous power p into

$$p = \mathbf{u} \cdot \mathbf{a} = -\mathbf{u} \cdot \nabla P + \mathbf{u} \cdot \mathbf{f} + \mathbf{u} \cdot \mathbf{D}, \quad (13)$$

where $-\nabla P$, \mathbf{f} , and \mathbf{D} are the pressure gradient, external forces, and dissipative forces, respectively. In 3D flows, the dissipative forces consist of the viscous forces alone, $\mathbf{D} = \nu \nabla^2 \mathbf{u}$; while for 2D flows, the dissipative forces include both viscous forces and friction forces, $\mathbf{D} = \nu \nabla^2 \mathbf{u} - \alpha \mathbf{u}$, where $\alpha > 0$ is the linear friction coefficient. Numerical simulation data show [87] that in both 2D and 3D flows, the magnitude of the pressure gradient term $-\mathbf{u} \cdot \nabla P$ overwhelms all other terms and determines the magnitude of p , but the contributions to the third moment of p are more subtle and show interesting differences between 2D and 3D flows.

In 2D flows, the pressure gradient term is also negatively skewed, and it contributes to nearly 2/3 of $\langle p^3 \rangle$, with the other dominant contribution being provided by the correlation between the pressure gradient and the friction, $\langle(-\mathbf{u} \cdot \nabla P)^2(-\alpha \mathbf{u} \cdot \mathbf{u})\rangle$. In 3D flows, the situation is completely different: the skewness of the pressure gradient term is very small, even slightly positive, so its direct contribution to $\langle p^3 \rangle$ is very small and of opposite sign, compared to $\langle p^3 \rangle$. The dominant term that contributes to $\langle p^3 \rangle$ is the cross term between the pressure gradient and the viscous forces, $\langle(-\mathbf{u} \cdot \nabla P)^2(\nu \mathbf{u} \cdot \nabla^2 \mathbf{u})\rangle$ [87].

Therefore, the pressure gradient term acts very differently in 2D and 3D flows. In 2D, it behaves according to naive expectation, insofar as it provides the dominant term of the fluctuations, and contributes significantly to the observed asymmetry of the distribution of p . In comparison, although the pressure term in 3D also provides the main contribution to the variance of p , it hardly provides any significant contribution to the observed asymmetry of the distribution of p .

Note that in homogenous flows, the pressure gradient term

alone does not change the total energy in the flow since $\langle \mathbf{u} \cdot \nabla P \rangle = 0$, i.e., the pressure gradient term merely redistributes kinetic energy within the flow. The different role played by the pressure gradient term would imply that the way of energy redistribution is different in 2D and 3D flows. Indeed, the averaged value of $-\mathbf{u} \cdot \nabla P$ conditioned on the kinetic energy of the particles reveals that in 2D flows, particles get as much energy from pressure gradient forces as they lose it, independently of their velocity: $\langle -\mathbf{u} \cdot \nabla P | \mathbf{u}^2 \rangle = 0$ for all \mathbf{u}^2 . In contrast, in 3D flows, the mean pressure contribution conditioned on the velocity is negative for particles with small velocities: $\langle -\mathbf{u} \cdot \nabla P | \mathbf{u}^2 \rangle < 0$ for $\mathbf{u}^2 \lesssim 2\langle \mathbf{u}^2 \rangle$, and it is positive for particles with large velocities. This implies that the pressure gradient term takes kinetic energy away from slow particles and gives it to fast particles [87]. Without other terms to stop this action, the pressure gradient term alone could potentially drive the flow into singularities. This observation might provide new insight into the long-standing Millennium problem on the regularity of the Navier-Stokes equations [88, 89].

The decomposition of the instantaneous power p in the form of Eq. (13) is certainly not unique. A possible alternative consists in decomposing the fluid acceleration into a local part $\mathbf{a}_L = \partial \mathbf{u} / \partial t$ and a convective part $\mathbf{a}_C = \mathbf{u} \cdot \nabla \mathbf{u}$. This decomposition separates the effect of the flow seen by the particle as resulting from the time variation of the velocity field locally, \mathbf{a}_L , and from the advection by a time-independent (frozen) flow, \mathbf{a}_C . It has been noticed that the two components \mathbf{a}_L and \mathbf{a}_C cancel each other to a large extent [90, 91]. How their contributions to the instantaneous power p behave and what they reveal about the irreversibility of the flow are interesting problems for future study.

The results discussed above are all for incompressible turbulence. As predicted by the study of the two-particle statistics governed by Burgers equation [46], in compressible flows, the irreversibility is expected to manifest itself in a stronger way. It would be interesting to confirm these predictions, by either experimental or numerical simulation data.

Another direction worth exploring concerns the effect of particle inertia on the effects discussed here for fluid particles. Particles whose densities differ from the fluid density or whose sizes are larger than the Kolmogorov scale do not follow the flow faithfully due to their inertia. There has been a wealth of literature on the dynamics of various inertia particles [19, 92–96]. It will be therefore interesting to carry out analyses similar to that in Refs. [84, 87] to see how the irreversibility of the flow is reflected in the dynamics of inertial particles.

IV. SUMMARY AND DISCUSSION

In this brief review, we discussed how Lagrangian statistics, obtained by following the motion of fluid particles in the flow, are sensitive to the intrinsic time irreversibility of turbulence. This irreversibility is a consequence of the property that, in turbulent flows, the kinetic energy is supplied into the fluid motion at a scale very different from the scale at which the en-

ergy is dissipated. In high-Reynolds number flows, where the forcing scales and the dissipation scales are widely separated, energy is transferred at a constant rate through scales, over a wide range of scales. This energy cascade and dissipation process are irreversible and the issue is how these phenomena affect the motion of fluid particles.

We emphasize that depending on the spatial dimension, the energy is transferred either to small scales or to large scales, to be dissipated either by viscosity (in 3D), or by friction (in 2D). This results in very different physical mechanisms. The investigation of multi-particle Lagrangian statistics, obtained by following sets of particles, allows the identification of at least one length scale. As a consequence, multi-particle Lagrangian statistics naturally sense the energy flux and reflect it in the change of kinetic energy associated with the relative motion, the dynamics of the perceived velocity gradients and the shape deformation of isotropic objects. Single-particle statistics, on the other hand, does not permit such an unambiguous identification of a length scale. Previous attempts to construct a length scale from single particle trajectories and hence to connect single-particle statistics with Eulerian statistics that depend on the energy flux are fundamentally questionable, and lead to incorrect predictions. While the statistics of the usual Lagrangian structure function do not permit to distinguish the arrow of time, the statistics of other quantities, such as the energy change following a fluid particle, do reveal the irreversibility of the flow. In this case, irreversibility is due to the energy dissipation, i.e., kinetic energy is eventually converted to thermal energy. Those statistics, therefore, show similar behavior in both 2D and 3D, despite the opposite directions of the energy flux in flows with different spatial dimensions.

Most of the work reviewed here originated from observations in physical experiments and numerical simulations that built on rapid progresses in experimental techniques and computational power. As more and more high-quality, high-resolution data are generated by the research community and are being shared with the whole community (e.g., in large public databases [97, 98]), we expect that other properties of Lagrangian statistics in turbulent flows will be discovered and their connection with the time irreversibility of the flow, or more generally, with the dynamical properties of the flow, be understood. From a broader theoretical point of view, fluid turbulence is a well-known example of out-of-equilibrium system. How to relate what we learned from studying fluid turbulence to other non-equilibrium systems is another open area for future investigation.

Acknowledgments

We thank G. Boffetta, G. Falkovich, N. Francois, R. Grauer, J. Jucha, M. Shats, and H. Xia for stimulating discussions during our work. We are grateful to the Max Planck Society for continuous support to our research. AP also acknowledges financial support from ANR (contract TEC 2), the Alexander von Humboldt Foundation, and the PSMN at the Ecole Normale Supérieure de Lyon.

-
- [1] L. D. Landau and E. M. Lifshitz. *Fluid Mechanics*. Butterworth-Heinemann, 1987.
 - [2] L. D. Landau and E. M. Lifshitz. *Statistical Physics*. Butterworth-Heinemann, 1980.
 - [3] B. Derrida. Non-equilibrium steady states: fluctuations and large deviations of the density and of the current. *J. Stat. Mech. Theor. Exp.*, 9:P07023, 2007.
 - [4] S. Ciliberto, S. Joubaud, and A. Petrosyan. Fluctuations in out-of-equilibrium systems: from theory to experiment. *J. Stat. Mech.*, 12:P12003, 2010.
 - [5] T. Tennekes and J. L. Lumley. *A First Course in Turbulence*. The MIT Press, Cambridge, USA, 1972.
 - [6] U. Frisch. *Turbulence: The Legacy of A. N. Kolmogorov*. Cambridge University Press, Cambridge, England, 1995.
 - [7] S. B. Pope. *Turbulent Flows*. Cambridge University Press, Cambridge, England, 2000.
 - [8] R. H. Kraichnan. Inertial ranges in two-dimensional turbulence. *Phys. Fluids*, 10:1417–1423, 1967.
 - [9] G. Boffetta and R. E. Ecke. Two-dimensional turbulence. *Annu. Rev. Fluid Mech.*, 44:417–451, 2012.
 - [10] H. A. Rose and P. L. Sulem. Fully developed turbulence and statistical mechanics. *J. Physique*, 39:441–484, 1978.
 - [11] A. N. Kolmogorov. The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers. *Dokl. Akad. Nauk SSSR*, 30:301–305, 1941.
 - [12] A. N. Kolmogorov. Dissipation of energy in the locally isotropic turbulence. *Dokl. Akad. Nauk SSSR*, 32:16–18, 1941.
 - [13] G. Falkovich, K. Gawedzki, and M. Vergassola. Particles and fields in fluid turbulence. *Rev. Mod. Phys.*, 73:913–975, 2001.
 - [14] J. Mann, S. Ott, and J. S. Andersen. Experimental study of relative, turbulent diffusion. Technical Report Risø-R-1036(EN), Risø National Laboratory, 1999.
 - [15] A. La Porta, G. A. Voth, A. M. Crawford, J. Alexander, and E. Bodenschatz. Fluid particle accelerations in fully developed turbulence. *Nature*, 409:1017–1019, 2001.
 - [16] N. Mordant, P. Metz, O. Michel, and J.-F. Pinton. Measurement of Lagrangian velocity in fully developed turbulence. *Phys. Rev. Lett.*, 87:214501, 2001.
 - [17] B. Lüthi, A. Tsinober, and W. Kinzelbach. Lagrangian measurements of vorticity dynamics in turbulent flow. *J. Fluid Mech.*, 528:87–118, 2005.
 - [18] M. Bourgoin, N. T. Ouellette, H. Xu, J. Berg, and E. Bodenschatz. The role of pair dispersion in turbulent flow. *Science*, 311:835–838, 2006.
 - [19] R. Volk, N. Mordant, G. Verhille, and J.-F. Pinton. Laser doppler measurement of inertial particle and bubble accelerations in turbulence. *Europhys. Lett.*, 81:34002, 2008.
 - [20] H. Xu, A. Pumir, and E. Bodenschatz. The pirouette effect in turbulent flows. *Nature Phys.*, 7:709–712, 2011.
 - [21] P. K. Yeung and S. B. Pope. Lagrangian statistics from direct numerical simulations of isotropic turbulence. *J. Fluid Mech.*, 207:531–586, 1989.
 - [22] A. Pumir, B. I. Shraiman, and M. Chertkov. Geometry of Lagrangian dispersion in turbulence. *Phys. Rev. Lett.*, 85:5324–5327, 2000.
 - [23] P. K. Yeung and M. S. Borgas. Relative dispersion in isotropic turbulence. Part 1. Direct numerical simulations and Reynolds-number dependence. *J. Fluid Mech.*, 503:93–124, 2004.
 - [24] L. Biferale, G. Boffetta, A. Celani, B. J. Devenish, A. Lanotte, and F. Toschi. Multiparticle dispersion in fully developed turbulence. *Phys. Fluids*, 17:111701, 2005.
 - [25] J. F. Hackl, P. K. Yeung, and B. L. Sawford. Multi-particle and tetrad statistics in numerical simulations of turbulent dispersion. *Phys. Fluids*, 23:065103, 2011.
 - [26] G. Falkovich, H. Xu, A. Pumir, E. Bodenschatz, L. Biferale, G. Boffetta, A. S. Lanotte, and F. Toschi. On lagrangian single-particle statistics. *Phys. Fluids*, 24:055102, 2012.
 - [27] L. F. Richardson. Atmospheric diffusion shown on a distance-neighbour graph. *Proc. Roy. Soc. Lond. A*, 110:709–737, 1926.
 - [28] B. L. Sawford. Turbulent relative dispersion. *Annu. Rev. Fluid Mech.*, 33:289–317, 2001.
 - [29] L. Biferale, G. Boffetta, A. Celani, B. J. Devenish, A. Lanotte, and F. Toschi. Lagrangian statistics of particle pairs in homogeneous isotropic turbulence. *Phys. Fluids*, 17:115101, 2005.
 - [30] B. L. Sawford, P. K. Yeung, and J. F. Hackl. Reynolds number dependence of relative dispersion statistics in isotropic turbulence. *Phys. Fluids*, 19:065111, 2008.
 - [31] J. P. L. C. Salazar and L. R. Collins. Two-particle dispersion in isotropic turbulent flows. *Annu. Rev. Fluid Mech.*, 41:405–432, 2009.
 - [32] F. Toschi and E. Bodenschatz. Lagrangian properties of particles in turbulence. *Annu. Rev. Fluid Mech.*, 41:375–404, 2009.
 - [33] R. Bitane, H. Homman, and J. Bec. Time scales of turbulent relative dispersion. *Phys. Rev. E*, 86:045302, 2012.
 - [34] G. L. Eyink. Stochastic flux freezing and magnetic dynamo. *Phys. Rev. E*, 83:056405, 2011.
 - [35] G. I. Taylor. Diffusion by continuous movements. *Proc. Lond. Math. Soc.*, 20:196–212, 1922.
 - [36] G. K. Batchelor. The application of the similarity theory of turbulence to atmospheric diffusion. *Q. J. R. Meteorol. Soc.*, 76:133–146, 1950.
 - [37] G. K. Batchelor. Diffusion in a field of homogeneous turbulence. II. The relative motion of particles. *Proc. Camb. Phil. Soc.*, 48:345–362, 1952.
 - [38] A. S. Monin and A. M. Yaglom. *Statistical Fluid Mechanics*, volume 2. MIT Press, Cambridge, MA, 1975.
 - [39] A. Pumir, B. I. Shraiman, and M. Chertkov. The Lagrangian view of energy transfer in turbulent flow. *Europhys. Lett.*, 56:379–385, 2001.
 - [40] H. Xu, N. T. Ouellette, and E. Bodenschatz. Evolution of geometric structures in intense turbulence. *New J. Phys.*, 10:013012, 2008.
 - [41] B. L. Sawford, P. K. Yeung, and M. S. Borgas. Comparison of backwards and forwards relative dispersion in turbulence. *Phys. Fluids*, 17:095109, 2005.
 - [42] J. Berg, B. Lüthi, J. Mann, and S. Ott. Backwards and forwards relative dispersion in turbulent flow: An experimental investigation. *Phys. Rev. E*, 74:016304, 2006.
 - [43] A. D. Bragg, P. J. Ireland, and L. R. Collins. Forward and backward in time dispersion of fluid and inertial particles in isotropic turbulence. *arXiv*, page 1403.5502, 2014.
 - [44] J. Jucha, H. Xu, A. Pumir, and E. Bodenschatz. Time-reversal-symmetry breaking in turbulence. *Phys. Rev. Lett.*, 113:054501, 2014.
 - [45] G. Falkovich and A. Frishman. Single flow snapshot reveals the future and the past of pairs of particles in turbulence. *Phys. Rev. Lett.*, 110:214502, 2013.
 - [46] A. Frishman and G. Falkovich. New type of anomaly in turbulence. *Phys. Rev. Lett.*, 113:024501, 2014.
 - [47] P. Castiglione and A. Pumir. Evolution of triangles in a two-dimensional turbulent flow. *Phys. Rev. E*, 64:056303, 2001.
 - [48] M. Chertkov, A. Pumir, and B. I. Shraiman. Lagrangian tetrad

- dynamics and the phenomenology of turbulence. *Phys. Fluids*, 11:2394–2410, 1999.
- [49] A. Pumir, E. Bodenschatz, and H. Xu. Tetrahedron deformation and alignment of perceived vorticity and strain in a turbulent flow. *Phys. Fluids*, 25:035101, 2013.
- [50] B. Lüthi, S. Ott, J. Berg, and J. Mann. Lagrangian multi-particle statistics. *J. Turbul.*, 8:45, 2007.
- [51] R. Betchov. An inequality concerning the production of vorticity in isotropic turbulence. *J. Fluid Mech.*, 1:497–504, 1956.
- [52] E. D. Siggia. Invariants for the one-point vorticity and strain rate correlation functions. *Phys. Fluids*, 24:1934–1936, 1981.
- [53] R. M. Kerr. Histograms of helicity and strain in numerical turbulence. *Phys. Rev. Lett.*, 59:783–786, 1987.
- [54] A. Tsinober, E. Kit, and T. Dracos. Experimental investigation of the field of velocity gradients in turbulent flows. *J. Fluid Mech.*, 242:169–192, 1992.
- [55] P. Vieillefosse. Internal motion of a small element of fluid in an inviscid flow. *Physica A*, 125:150–162, 1984.
- [56] B. J. Cantwell. Exact solution of a restricted Euler equation for the velocity gradient tensor. *Phys. Fluids A*, 4:782–793, 1992.
- [57] C. Meneveau. Lagrangian dynamics and models of the velocity gradient tensor in turbulent flows. *Annu. Rev. Fluid Mech.*, 43:219–245, 2011.
- [58] E. D. Siggia. Numerical study of small-scale intermittency in three-dimensional turbulence. *J. Fluid Mech.*, 107:375–406, 1981.
- [59] W. T. Ashurst, A. R. Kerstein, R. M. Kerr, and C. H. Gibson. Alignment of vorticity and scalar gradient with strain rate in simulated Navier-Stokes turbulence. *Phys. Fluids*, 30:2343–2353, 1987.
- [60] A. Tsinober. *An Informal Conceptual Introduction to Turbulence*. Springer, Berlin, 2009.
- [61] A. Pumir and A. Naso. Insight on turbulent flows from Lagrangian tetrads. *C. R. Physique*, 13:889–898, 2012.
- [62] L. Chevillard and C. Meneveau. Lagrangian time correlations of vorticity alignments in isotropic turbulence: observations and model predictions. *Phys. Fluids*, 23:101704, 2011.
- [63] A. Pumir and M. Wilkinson. Orientation statistics of small particles in turbulence. *New J. Phys.*, 13:093030, 2011.
- [64] R. Ni, N. T. Ouellette, and G. A. Voth. Alignment of vorticity and rods with Lagrangian fluid stretching in turbulence. *J. Fluid Mech.*, 743:R3, 2014.
- [65] R. Ni, S. Kramel, N. T. Ouellette, and G. A. Voth. Measurements of the coupling between the tumbling of rods and the velocity gradient tensor in turbulence. *J. Fluid Mech.*, 766:202–225, 2015.
- [66] R. Zimmermann, Y. Gasteuil, M. Bourgoïn, R. Volk, A. Pumir, and J.-F. Pinton. Rotational intermittency and turbulence induced lift experienced by large particles in a turbulent flow. *Phys. Rev. Lett.*, 106:154501, 2011.
- [67] S. Klein, M. Gibert, A. Berut, and E. Bodenschatz. Simultaneous 3d measurement of the translation and rotation of finite-size particles and the flow field in a fully developed turbulent water flow. *Meas. Sci. Technol.*, 24:024006, 2013.
- [68] G. Bellani and E. A. Variano. Slip velocity of large neutrally buoyant particles in turbulent flows. *New J. Phys.*, 14:125009, 2012.
- [69] L. Chevillard and C. Meneveau. Orientation dynamics of small, triaxial-ellipsoidal particles in isotropic turbulence. *J. Fluid Mech.*, 737:571–596, 2013.
- [70] K. Gustavsson, J. Einarsson, and B. Mehlig. Tumbling of small axisymmetric particles in random and turbulent flows. *Phys. Rev. Lett.*, 112:014501, 2014.
- [71] G. G. Marcus, S. Parsa, S. Kramel, R. Ni, and G. A. Voth. Measurements of the solid-body rotation of anisotropic particles in 3d turbulence. *New J. Phys.*, 16:102001, 2014.
- [72] B. J. Devenish. Geometrical properties of turbulent dispersion. *Phys. Rev. Lett.*, 110:064504, 2013.
- [73] B. L. Sawford, S. B. Pope, and P. K. Yeung. Gaussian Lagrangian stochastic models for multi-particle dispersion. *Phys. Fluids*, 25:055101, 2013.
- [74] P. K. Yeung. Lagrangian investigations of turbulence. *Annu. Rev. Fluid Mech.*, 34:115–142, 2002.
- [75] L. Biferale, G. Boffetta, A. Celani, B. J. Devenish, A. Lanotte, and F. Toschi. Multifractal statistics of Lagrangian velocity and acceleration in turbulence. *Phys. Rev. Lett.*, 93:064502, 2004.
- [76] H. Xu, M. Bourgoïn, N. T. Ouellette, and E. Bodenschatz. High order Lagrangian velocity statistics in turbulence. *Phys. Rev. Lett.*, 96:024503, 2006.
- [77] K. P. Zybin and V. A. Sirota. Lagrangian and Eulerian velocity structure functions in hydrodynamic turbulence. *Phys. Rev. Lett.*, 104:154501, 2010.
- [78] G.-W. He. Anomalous scaling for Lagrangian velocity structure functions in fully developed turbulence. *Phys. Rev. E*, 83:025301, 2011.
- [79] M. S. Borgas. The multifractal Lagrangian nature of turbulence. *Phil. Trans. R. Soc. Lond. A*, 342:379–411, 1993.
- [80] L. Chevillard, S. G. Roux, E. Leveque, N. Mordant, J.-F. Pinton, and A. Arneodo. Lagrangian velocity statistics in turbulent flows: effects of dissipation. *Phys. Rev. Lett.*, 91:214502, 2003.
- [81] L. Chevillard, B. Castaing, A. Arneodo, E. Leveque, J.-F. Pinton, and S. G. Roux. A phenomenological theory of Eulerian and Lagrangian velocity fluctuations in turbulent flows. *C. R. Physique*, 13:899–928, 2012.
- [82] A. N. Kolmogorov. A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number. *J. Fluid Mech.*, 13:82–85, 1962.
- [83] B. L. Sawford and P. K. Yeung. Kolmogorov similarity scaling for one-particle Lagrangian statistics. *Phys. Fluids*, 23:091704, 2011.
- [84] H. Xu, A. Pumir, G. Falkovich, E. Bodenschatz, M. Shats, H. Xia, N. Francois, and G. Boffetta. Flight-crash events in turbulence. *Proc. Natl. Acad. Sci. USA*, 111(21):7558–7563, 2014.
- [85] N. Mordant. *Mesure lagrangienne en turbulence: mise en œuvre et analyse*. PhD thesis, Ecole Normale Supérieure de Lyon, 2001.
- [86] E. Leveque and A. Naso. Introduction of longitudinal and transverse Lagrangian velocity increments in homogeneous and isotropic turbulence. *Europhys. Lett.*, 108:54004, 2014.
- [87] A. Pumir, H. Xu, G. Boffetta, G. Falkovich, and E. Bodenschatz. Redistribution of kinetic energy in turbulent flows. *Phys. Rev. X*, 4:041006, 2014.
- [88] Jean Leray. Sur le mouvement d’un fluide visqueux emplissant l’espace. *Acta Mathematica*, 63:193–248, 1934.
- [89] C. L. Fefferman. *The millennium prize problems*, chapter Existence and smoothness of the Navier-Stokes equation, pages 57–67. Clay Mathematics Institute, Cambridge, MA, 2006.
- [90] A. Tsinober, P. Vedula, and P. K. Yeung. Random Taylor hypothesis and the behavior of local and convective accelerations in isotropic turbulence. *Phys. Fluids*, 13:1974–1984, 2001.
- [91] G. Gulitski, M. Kholmyansky, W. Kinzelbach, B. Lüthi, A. Tsinober, and S. Yorish. Velocity and temperature derivatives in high-Reynolds-number turbulent flows in the atmospheric surface layer. part 2. accelerations and related matters. *J. Fluid Mech.*, 589:83–102, 2007.
- [92] G. A. Voth, A. La Porta, A. M. Crawford, J. Alexander, and

- E. Bodenschatz. Measurement of particle accelerations in fully developed turbulence. *J. Fluid Mech.*, 469:121–160, 2002.
- [93] J. Bec, L. Biferale, G. Boffetta, A. Celani, M. Cencini, A. Lanotte, S. Musacchio, and F. Toschi. Acceleration statistics of heavy particles in turbulence. *J. Fluid Mech.*, 550:349–358, 2006.
- [94] N. M. Qureshi, M. Bourgoïn, C. Baudet, A. Cartellier, and Y. Gagne. Turbulent transport of material particles: An experimental study of finite size effects. *Phys. Rev. Lett.*, 99:184502, 2007.
- [95] H. Xu and E. Bodenschatz. Motion of inertial particles with size larger than kolmogorov scale in turbulent flows. *Physica D*, 237:2095–2100, 2008.
- [96] R. Zimmermann, L. Fiabane, Y. Gasteuil, R. Volk, and J.-F. Pinton. Characterizing flows with an instrumented particle measuring lagrangian accelerations. *New J. Phys.*, 15:015018, 2013.
- [97] Y. Li, E. Perlman, M. Wan, Y. Yang, C. Meneveau, R. Burns, S. Chen, A. Szalay, and G. L. Eyink. A public turbulence database cluster and applications to study Lagrangian evolution of velocity increments in turbulence. *J. Turbul.*, 9:31, 2008.
- [98] H. Yu, K. Kanov, E. Perelman, J. Graham, E. Frederix, R. Burns, A. Szalay, G. Eyink, and C. Meneveau. Studying Lagrangian dynamics of turbulence using on-demand fluid particle tracking in a public turbulence database. *J. Turbul.*, 13:N12, 2012.